Control Flow Analysis for Recursion Removal

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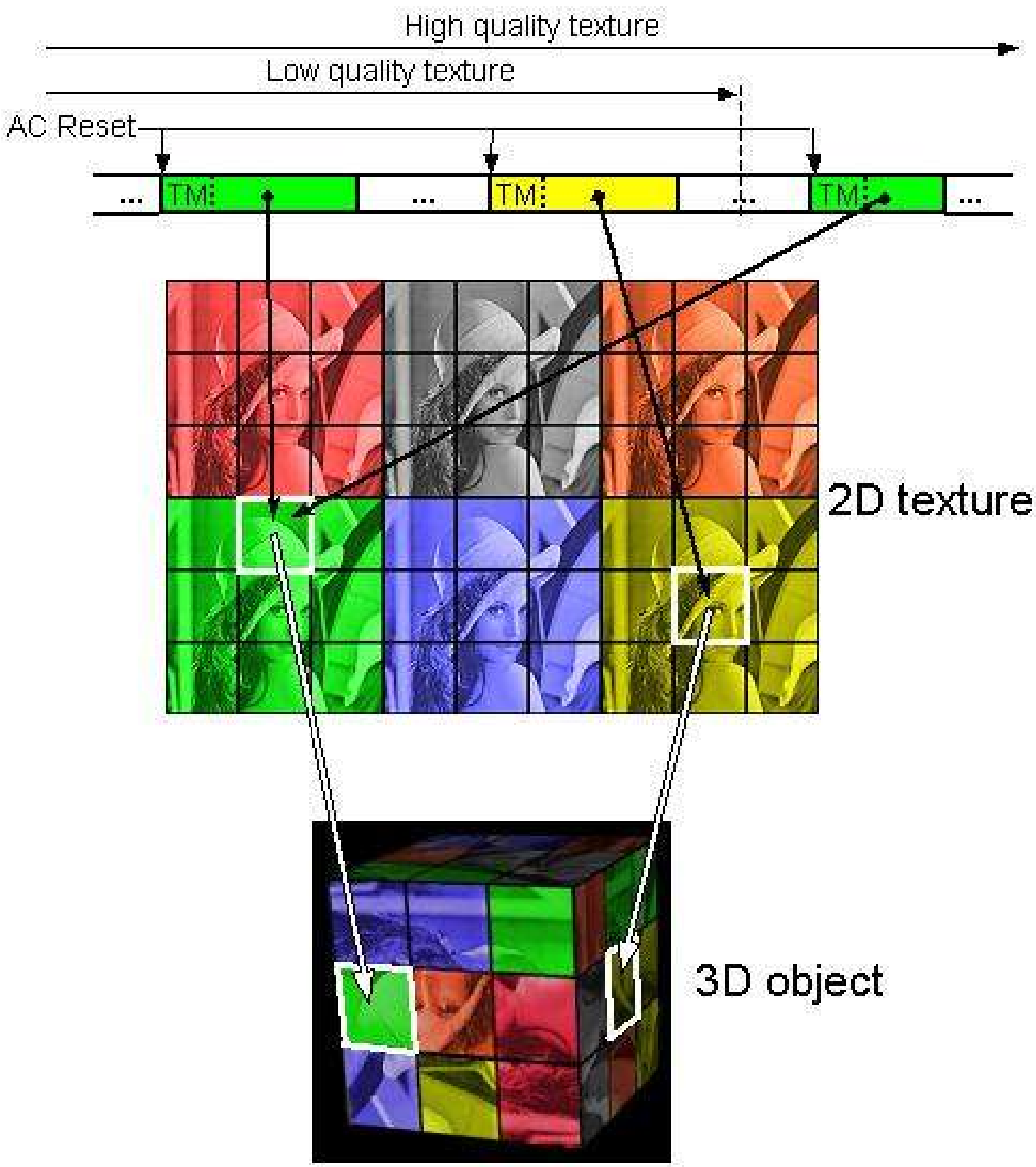
Introduction

- Recursion removal
  - Traditionally done to reduce resource consumption (time and memory)
  - Now meant as enabling step for other transformations, typically on imperative or OO code (C/C++/JAVA/...)
    - Usually dependency removal transformations, and parallelising transformations
    - Hence, main goals:
      - Try to introduce as little new dependencies in iterative resulting algorithm implementation as possible
      - Guarantee correctness in presence of side-effects
    - However, it is still useful to evaluate impact on execution time, memory usage

Visual texture coding



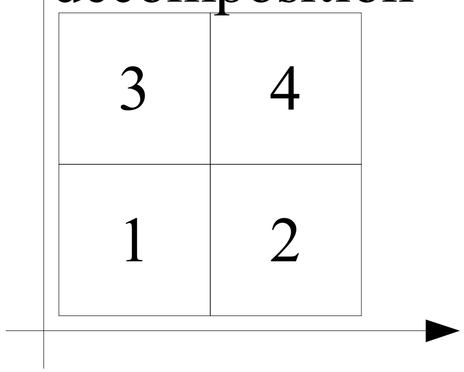
AC = Arithmetic Coder
TM = Texture marker

- Part of the MPEG-4 standard
- Transmission of still images
- Scalable
  - Quality: successive quantisation
  - Resolution: Wavelet based
  - Region of Interest (ROI) selection

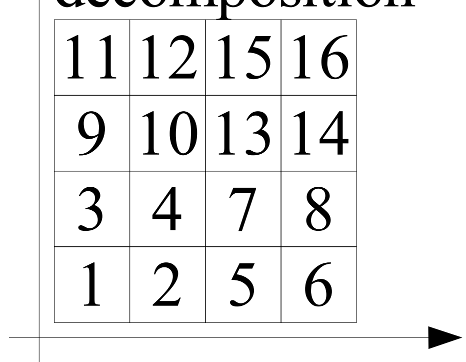
Recursive part of the VTC algorithm

- MPEG 4 VTC

1-level quad tree decomposition



2-level quad tree decomposition



VTC algorithm (partial)

```
Decode (d, x, y) {
  if (d= =0) {
    DecodePixel(x,y);
  } else {
    --d; k=1<<d;
    Decode(d,x,y);
    if (d= =4) Check();
    Decode(d,x+k,y);
    if (d= =4) Check();
    Decode(d,x,y+k);
    if (d= =4) Check();
    Decode(d,x+k,y+k);
    if (d= =4) Check();
}
```

Recursive part of the VTC algorithm

- Time spent in this part of VTC is 50% of overall time
- DecodePixel is expensive compared to argument trafo and recursive function calls
- Between recursive calls extra functionality is present
  - It has side-effects
  - It must be preserved

VTC algorithm (partial)

```
Decode (d, x, y) {
  if (d= =0) {
    DecodePixel(x,y);
  } else {
    --d; k=1<<d;
    Decode(d,x,y);
    if (d= =4) Check();
    Decode(d,x+k,y);
    if (d= =4) Check();
    Decode(d,x,y+k);
    if (d= =4) Check();
    Decode(d,x+k,y+k);
    if (d= =4) Check();
}
```

Recursive part of the VTC algorithm

- First step:
  - DecodePixel is expensive
  - DecodePixel is activated with argument values that are transformed through recursion

  Hence
  - First collect argument values, store in memory
  - Then use in iteration with DecodePixel calls

VTC algorithm (partial)

```
Decode (d, x, y) {
  if (d= =0) {
    DecodePixel(x,y);
  } else {
    --d; k=1<<d;
    Decode(d,x,y);
    if (d= =4) Check();
    Decode(d,x+k,y);
    if (d= =4) Check();
    Decode(d,x,y+k);
    if (d= =4) Check();
    Decode(d,x+k,y+k);
    if (d= =4) Check();
}
```

Recursive part of the VTC algorithm

- Second step:
  - Storing collected values in memory is expensive

  Hence

  - Replace recursive information collection with a formula producing these same argument values

VTC algorithm (partial)

```
Decode (d, x, y) {
  if (d= =0) {
    DecodePixel(x,y);
  } else {
    --d; k=1<<d;
    Decode(d,x,y);
    if (d= =4) Check();
    Decode(d,x+k,y);
    if (d= =4) Check();
    Decode(d,x,y+k);
    if (d= =4) Check();
    Decode(d,x+k,y+k);
    if (d= =4) Check();
}
```

Step 1: separate base case calculation from recursion

- Record all information that is needed in base case, in the original recursive algorithm without base case calculation
  - Applied to VTC:
    - Leave out call to DecodePixel
    - Add recording of d, x, y values in algorithm
- Loop over all collected information
  - Each time calling DecodePixel with correct argument values retrieved from memory
  - This loop is fully parallelizable, if the side-effects inside DecodePixel do not interfere

Pseudo-code result

VTC algorithm (info collection)

```
Decode (d, x, y) {
  if (d= =0) {
    info[++cnt]=(x,y);
  } else {
    --d; k=1<<d;
    Decode(d,x,y);
    if (d==4) deq4[cnt]=1;
    Decode(d,x+k,y);
    if (d==4) deq4[cnt]=1;
    Decode(d,x,y+k);
    if (d==4) deq4[cnt]=1;
    Decode(d,x+k,y+k);
    if (d==4) deq4[cnt]=1;
}
```

VTC algorithm (iterative part)

```
ItDecode(d,x,y) {
  Decode(d,0,0);
  for (i=0;i<cnt;i++) {
    DecodePixel(info[i]);
    if (deq4[i])
      Check();
  }
}
```

What has happened up to now?

- Recursive part of algorithm
  - Collects information
  - No longer dominates execution time
- Iterative part of algorithm
  - Uses collected information
  - Method itself does not impose inter-iteration dependencies
    - Hence: opportunity for parallelization
- Drawback
  - Storing all information needs a lot of memory, certainly a lot more than was needed in the recursion; bad for energy consumption

Step 2: replace recursive info collection with iterative info generation

- Determine iteration bound
  - Call to Decode(d,_,_) results in 4 calls to Decode(d-1,_,_)
  - Number of iterations easily found by solving recurrence equation

$$I(d) = 4\,I(d-1) \text{ with } I(0) = 1$$

  - With solution

$$I(d) = 4^d$$

Step 2: replace recursive info collection with iterative info generation

- Modeling of argument transformations
  - Call to Decode(d,x,y) results in 4 calls:
    Decode(d-1, x , y);
    Decode(d-1, x+k, y);
    Decode(d-1, x , y+k);
    Decode(d-1, x+k, y+k);

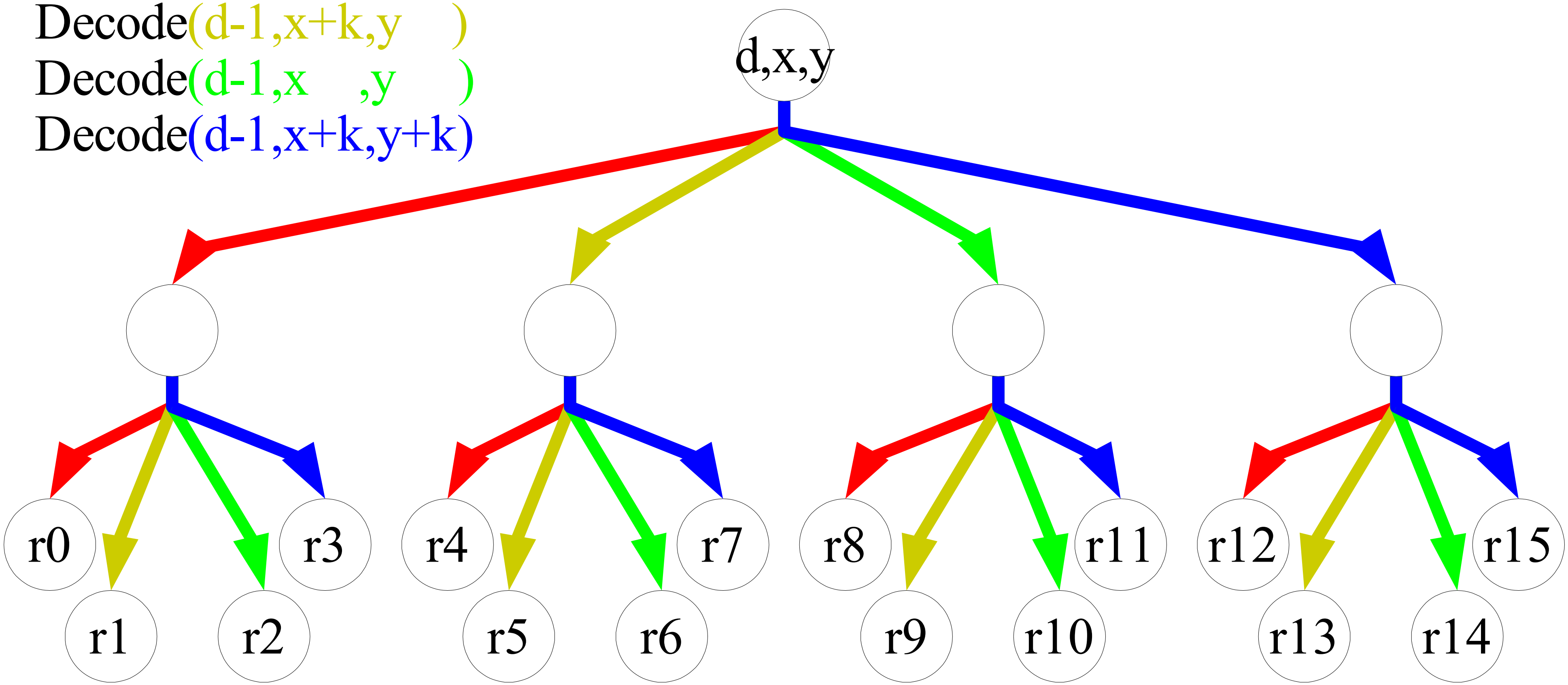
    (with k = 2^(d-1))

  - Before base case is reached, arguments have undergone a large amount of additions with different k-values = argument transformations

Call tree

Example: call to Decode(2,x,y) results in 4 calls:

Decode(d-1,x ,y)
Decode(d-1,x+k,y)
Decode(d-1,x ,y)
Decode(d-1,x+k,y+k)



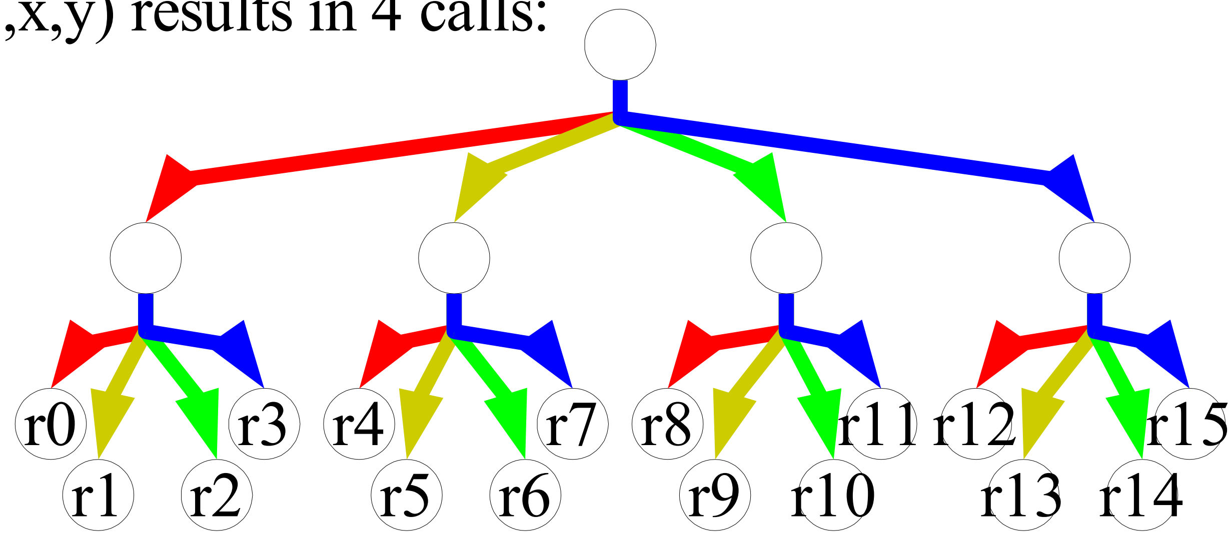
Problem:

How is (d,x,y) transformed through the recursion when reaching the base case ?

Call tree

Example: call to Decode(2,x,y) results in 4 calls:

Decode(d-1,x ,y)
Decode(d-1,x+k,y)
Decode(d-1,x ,y)
Decode(d-1,x+k,y+k)



Answer:
(for mathematical derivation: see paper)

$$r_j = \begin{bmatrix} 0 \\ \sum_{i=0}^{n-1} 2^i \left(\left(j \operatorname{div} 2^{2i} \right) \bmod 2 \right) \\ \sum_{i=0}^{n-1} 2^i \left(\left(j \operatorname{div} 2^{2i+1} \right) \bmod 2 \right) \end{bmatrix}$$

What has happened up to now?

VTC algorithm (info collection)

```
Decode (d, x, y) {
  if (d= =0) {
    info[++cnt]=(x,y);
  } else {
    --d; k=1<<d;
    Decode(d,x,y);
    if (d==4) deq4[cnt]=1;
    Decode(d,x+k,y);
    if (d==4) deq4[cnt]=1;
    Decode(d,x,y+k);
    if (d==4) deq4[cnt]=1;
    Decode(d,x+k,y+k);
    if (d==4) deq4[cnt]=1;
}
```

- Formula for rj can be used to replace non-grayed out part of VTC with iteration
  - Synthesize a loop that calculates rj values for j = 0 .. 4^d
- We have ignored the extra functionality between the recursive Decode calls
- Let's put it back in !

Handling intermediate functionality

- Problem: when must activation of

  "if (d==4) Check()"

  be scheduled, to preserve correctness in presence of side-effects inside Check() function ?

- Strategy:
  - We look for a relation R(j) between iteration counter j (which calculates successive rj's) and the result of condition (d == 4)
  - Then iterative algorithm calculates rj, followed by checking condition R(j) and possible activation of intermediate functionality

Handling intermediate functionality

- Consider call to Decode(4,_,_)
  - This results in 4 calls to Decode(3,_,_)
  - Each of these 4 calls results in 4 calls to Decode (2,_,_) => 4*4 = 16 calls
  - ...
  - Results in 4*4*4*4 = 256 calls to Decode(0,_,_)
    (call to Decode(0,_,_) is what activates base case calculation!)
- Conclusion:
  - Between two calls to Decode(4,_,_) there are 4^4 = 256 calls to Decode(d,_,_) with d < 4
  - Hence (d == 4) is true if j is multiple of 256 !
  - Similarly (d > 4) is true if j is multiple of 1024

Handling intermediate functionality

- Hence the following relation has been found

$$d = 4$$
$$\Leftrightarrow$$
$$j \; mod \; 256 = 0 \wedge j \; mod \; 1024 \neq 0$$

- Because 256 and 1024 are powers of two, this is cheaply implemented by bit masking

Resulting iterative VTC core

- Iterative version

for j = 0 to $4^n - 1$ {

  // calculate argument values

$$x\_bc = \sum_{i=0}^{n-1} 2^i((j \text{ div } 2^{2i}) \bmod 2);$$

$$y\_bc = \sum_{i=0}^{n-1} 2^i((j \text{ div } 2^{2i+1}) \bmod 2);$$

  // activate base case calculation

  DecodePixel(x_bc,y_bc);

  // activation intermediate functionality

  if $((j \bmod 256 = 0) \&\& (j \bmod 1024 \neq 0))$

    Check();

}

Intermediate conclusions

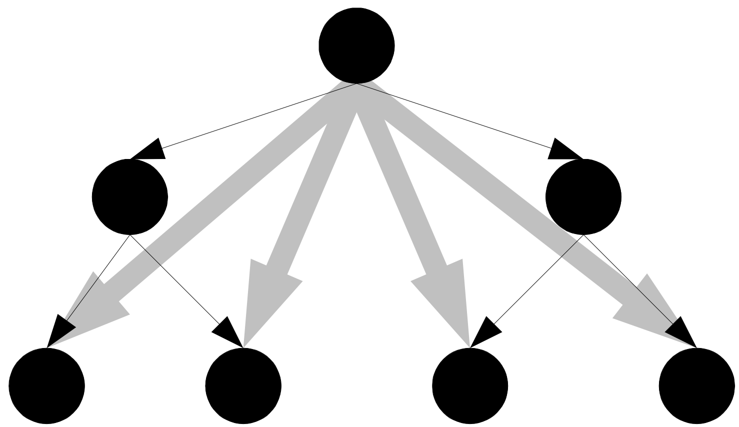
- What have we done up to now?
  - Separate base case calculation from recursion
    - To isolate dominant part from recursion
  - Replace argument recording with argument calculation
    - To remove argument recording memory usage
    - Solution to three sub-problems was required
      - Determination of iteration bound
      - Modeling argument transformations through the recursion
      - Activation of intermediate functionality at correct moments in time

Intermediate conclusions

- How general is this approach ?
  - Has been generalised and extended to handle recursive algorithms with any combination of
    - Multiple base cases
    - Multiple recursive cases
    - Intermediate functionality which depends on any combination of recursive function arguments (or other arguments, e.g. global variables)
    - Any kind of argument transformation functions
    - Functions with return values and functions to combine return values to new return values
  - In the most general case all functions may have side-effects
  - So, it is quite general!

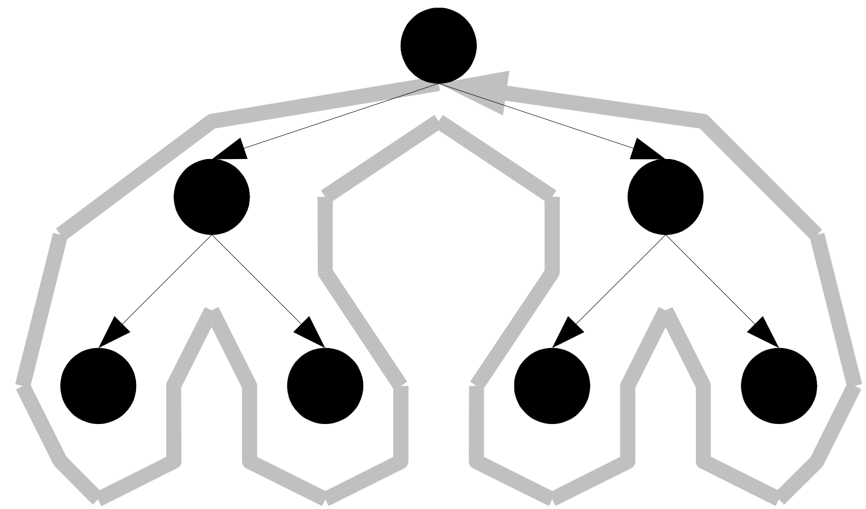
Comparison iterative - recursive

- Iterative algorithm



- Traverses each path from root node to leaf
- Arrows closer to root are evaluated more often than arrows nearer to leaf

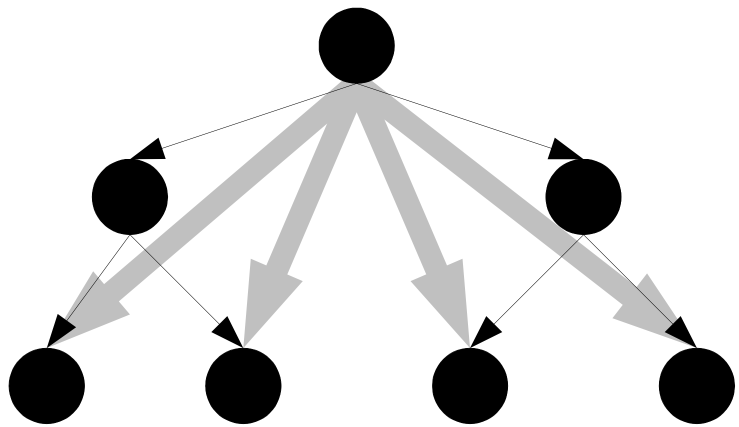
- Recursive algorithm



- Performs depth-first traversal of call tree
- Each arrow is evaluated exactly once

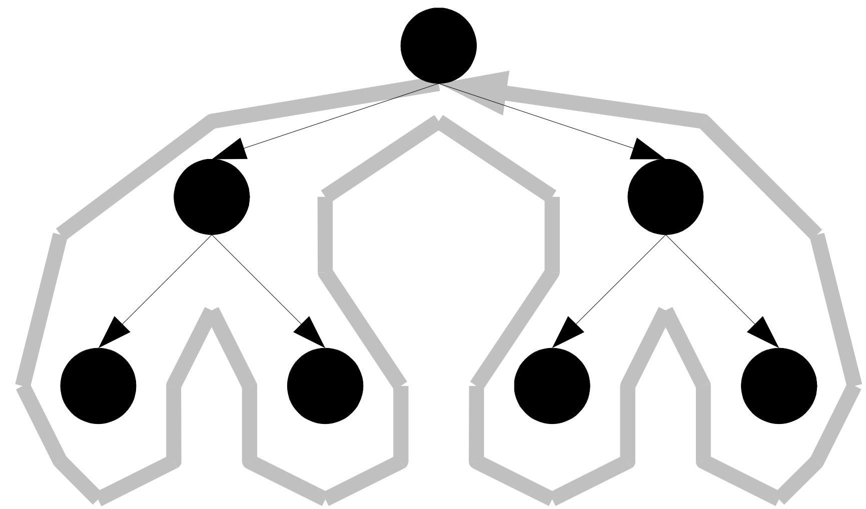
Comparison calculation requirements

- Iterative algorithm



- Each arrow represents
  - Function call
  - Argument trafo step
- There are (at most) $\frac{B^{D_{MAX}+1}-1}{B-1}-1$ arrows

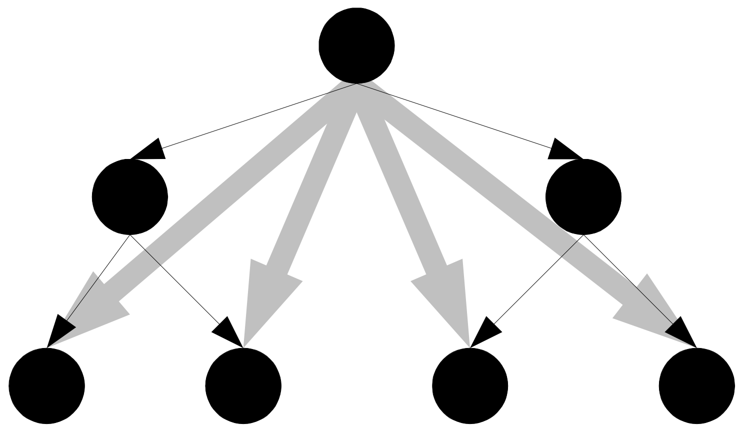
- Recursive algorithm



- Each arrow represents
  - Argument trafo step
- Arrow from depth d to depth d+1 is redone (at most) $B^{D_{MAX}-1}$ times
- From depth d to d+1 there are B^{d+1} arrows
- Total #calcs = $D_{MAX}\, B^{D_{MAX}}$

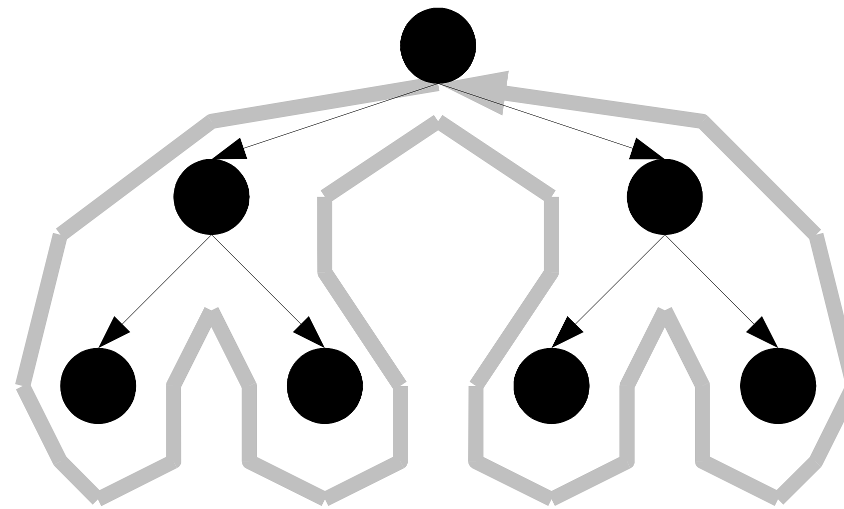
Comparison memory requirements

- Iterative algorithm



- Some fixed amount of book keeping memory required
- No extra memory depending on recursion depth

- Recursive algorithm



- Memory required is at most

  = stack frame size X recursion depth

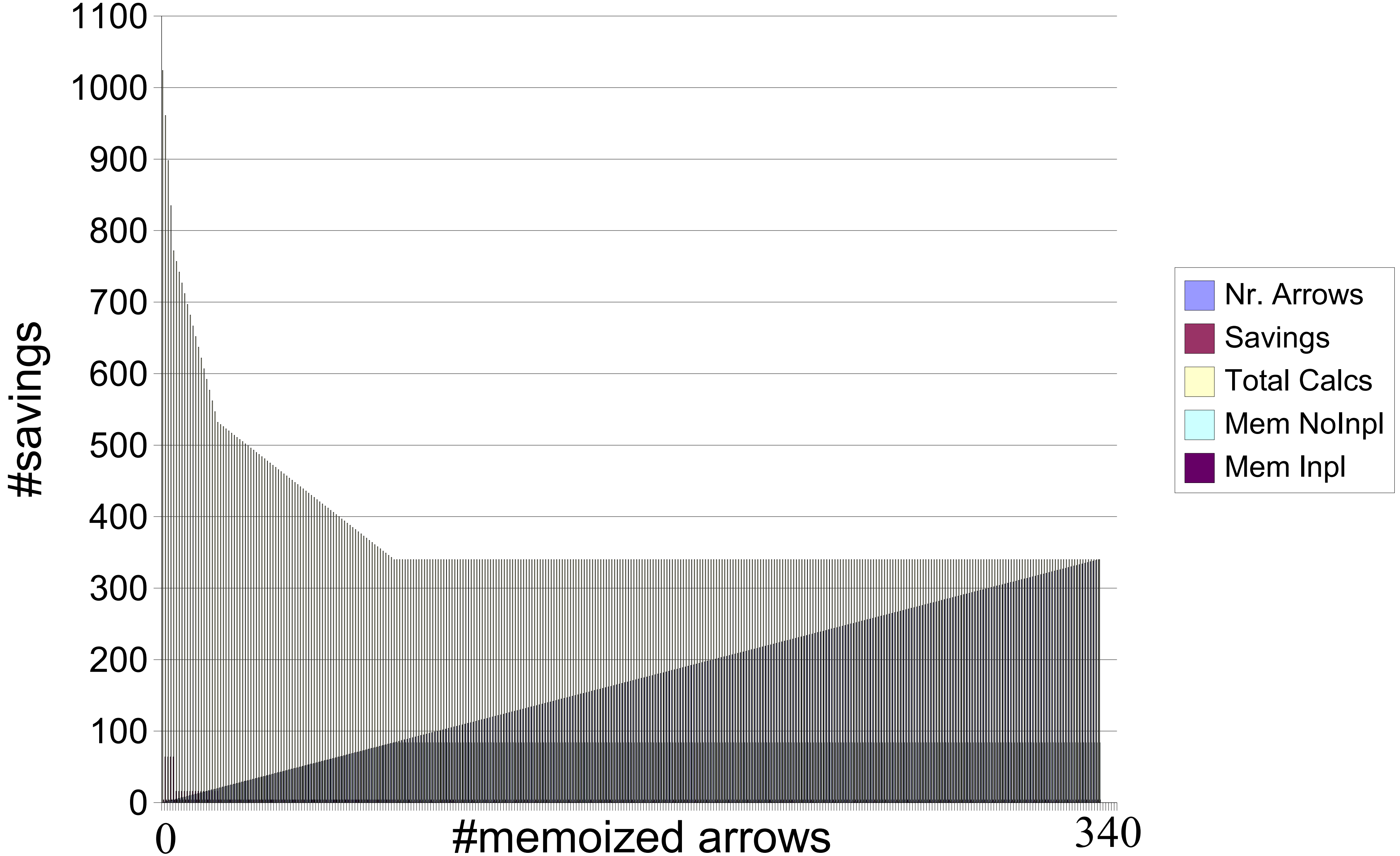
Systematic trade-offs

- First trade-off:
  - Trade-off argument transformation calculations verus memory used
  - By applying partial memoization
- Second trade-off
  - Trade-off memoization memory for parallelizability
  - By applying in-place optimization

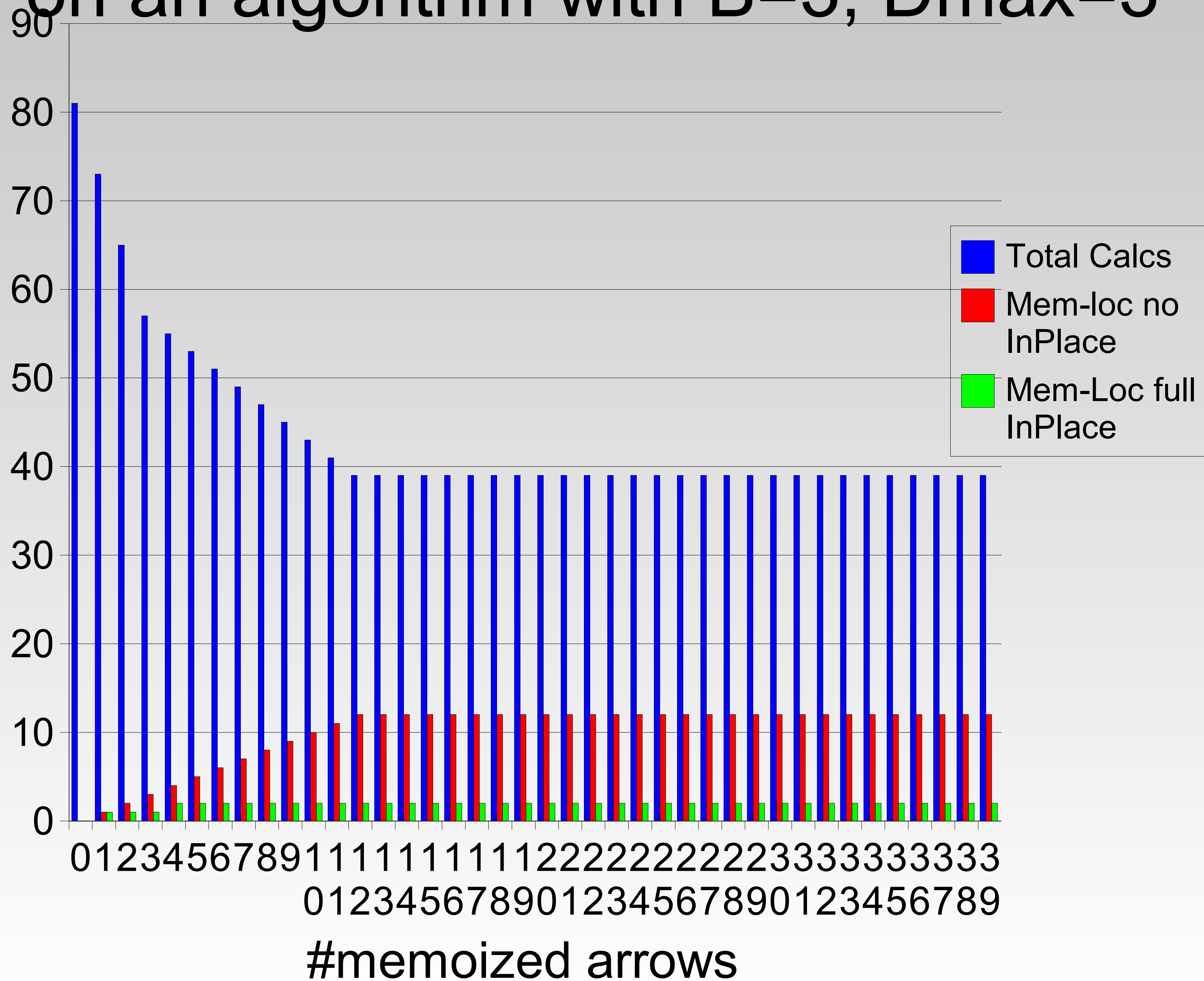
First trade-off: calculations vs. memory

- Storing results from argument transformation step going from depth d to d+1 reduces number of argument transformation steps with an amount of $B^{D_{MAX}-d}-1$
- Memoization of leaf does not have advantage (leaf is needed only once, $B^{D_{MAX}-D_{MAX}}-1=0$)
- Storing each result in separate memory location
  - Reduces amount of calculations to amount in recursive algorithm
  - But uses up to $\dfrac{B^{D_{MAX}+1}-1}{B-1}-1$ memory locations
  - Recursive version needs only D_{max} locations
- This version has potentially full parallelisability, but this is usually not needed

Effect of memoization on #calcs, #mem



Effect of memoization on #calcs, #mem on an algorithm with B=3, Dmax=3



Second trade-off: memory vs. parallelisability

- Not all argument transformation results are needed at the same time (unless fully parallelised version is desired)
- Hence memory locations can be reused
- By full memoization and fully exploiting limited life-times
  - Can achieve same time and memory complexity as recursive algorithm or better
  - This depends on such factors as
    - Known lack of side-effects in certain parts of the algorithm
    - Possibilities for symbolically simplifying r_j - equation

Summary of trade-offs

	Recursive calls	Argument trafo calcs	Memory locations	Parallel. oportunities
Recursive version	$\frac{B^{D_{MAX}+1}-1}{B-1}-1$	$\frac{B^{D_{MAX}+1}-1}{B-1}-1$	$S.D_{MAX}$	none
Iterative version	0	$D_{MAX}\,B^{D_{MAX}}$	0	full
Iterative + partial memoization	0	Between $D_{MAX}\,B^{D_{MAX}}$ and $\frac{B^{D_{MAX}+1}-1}{B-1}-1$	Between 0 and $\frac{B^{D_{MAX}+1}-1}{B-1}-1$	full
Iterative + memoization + in-place	0	Between $D_{MAX}\,B^{D_{MAX}}$ and $\frac{B^{D_{MAX}+1}-1}{B-1}-1$	Full memo: between $\frac{B^{D_{MAX}+1}-1}{B-1}-1$ and $S'.D_{MAX}$	Between full and none

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Summary and future work

- Method for removing and analysing recursion in quite general cases
  - Here applied on VTC algorithm
  - Many other examples have been transformed as well
- Recursion removal on VTC enabled further transformation to get energy reduction of up to factor 2 (without reduction of execution time). For details, see:

Zhe Ma, Chun Wong, Stefaan Himpe, Eric Delfosse, Francky Catthoor and Geert Deconinck, "Task concurrency analysis and exploration of Visual Texture Coder on a Heterogeneous Platform", in: Proceedings of the 2003 IEEE workshop on signal processing systems (SIPS03)

The End...

...thank you!